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Demand, Capacity and Queues – Revisiting Queueing Theory

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Queueing theory is the study of the relationship between demand, service capacity and waiting times using mathematical models. We introduced this Operations Research topic in an earlier issue this year. The analytical tools are useful for capacity sizing and impact analysis of operational interventions. There are many queueing models available, and we will talk about one of them called a blocking model. The blocking model is used specifically, when even some waiting is not acceptable. This model describes a queueing process where customers arrive at a random rate, treatment duration is quite varied and there are a number of service providers to cater for the patients, be they beds or diagnostic machines.

Case study: Sizing the number of recovery beds after endoscope procedure

With increasing patient workload, the Endoscopy Centre of Tan Tock Seng Hospital planned to expand the number of Endoscopy theatres from 4 to 6. Patients given sedation during the procedure require post-recovery beds and close monitoring. Hence sufficient bed capacity would be essential for patient safety. Here we want to compute the number of post-recovery beds needed given the planned endoscopy theatre expansion, and we do not want the patients to be waiting for a recovery bed.

How do we apply queueing theory here?

Applying the blocking model to this, the patients needing the recovery beds (servers) arrive according to a Poisson process. Their service times (recovery times) can follow any general probability distribution function. There are "c" servers (recovery beds) available. Each newly arriving customer immediately goes into service (recovery) if there is a server (recovery bed) available, and the patient has to wait if all servers are occupied. This queueing system is labelled as a M/G/c/c loss system.

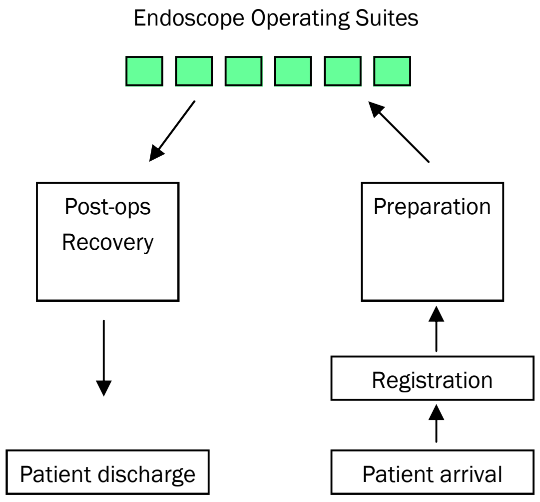

Figure 1: Patient flow in endoscopy suite

Data, method and results

Five days of data (318 records) were studied to analyze endoscopy patients' arrival rates, procedure and recovery time and percentages of patients on sedation. Sedated patients coming out from operating theatres formed the "demand" for the post-recovery beds. There were 4 endoscopy theatres and had to be expanded to 6.

Prob (%) of not finding a bed after Ops	Demand = # of theatres * %req beds * Recovery time / Ops time									
# of Recovery beds	5	6	7	8	9	10	11	12	13	14
22	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.7	1.2
21	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.6	1.1	2.0
20	0.0	0.0	0.0	0.0	0.1	0.2	0.5	1.0	1.8	3.0
19	0.0	0.0	0.0	0.0	0.1	0.4	0.8	1.6	2.8	4.4
18	0.0	0.0	0.0	0.1	0.3	0.7	1.5	2.7	4.3	6.3
17	0.0	0.0	0.1	0.2	0.6	1.3	2.5	4.1	6.2	8.6
16	0.0	0.0	0.1	0.5	1.1	2.2	3.9	6.0	8.6	11.5
15	0.0	0.1	0.3	0.9	2.0	3.6	5.9	8.6	11.6	14.8
14	0.0	0.2	0.7	1.7	3.4	5.7	8.5	11.7	15.1	18.6
13	0.1	0.5	1.4	3.1	5.4	8.4	11.9	15.5	19.2	22.8
12	0.3	1.1	2.7	5.1	8.3	12.0	15.9	19.9	23.7	27.5
11	0.8	2.3	4.8	8.1	12.1	16.3	20.6	24.8	28.7	32.4
10	1.8	4.3	7.9	12.2	16.8	21.5	26.0	30.2	34.1	37.7

Table 1: Percentage of patients who will not find a recovery bed

The duration of each procedure, number of theatres and percentage of patients on sedation would then determine the demand for post-recovery beds. The queueing model was used with different scenarios to compute the probability that a patient could not find a bed, post-procedure. This model was validated with current data.

How to use the look-up table for "loss" probability (See Table 1)

For instance, assume 6 theatres are in operation, 50% of patients are sedated, average recovery needs 90 min, and procedure takes 30 min. Then we will read off under column heading "9" (6*0.5*90/30). With 16 recovery beds (row heading), we will expect 1.1% of the sedated patients will not find an empty recovery bed after the procedure. The table is colour coded by probability of "loss" (not finding a bed) for easy referencing.

Model validation

A blocking queueing model only requires the arrival process to be approximately Poisson, while the service rate can be general. The coefficient of variance (given by standard deviation divided by mean) of the inter-arrival times was found to be very close to 1 (0.96). This validated the use of this queueing model.

Insights from the queueing analysis

1. **Planning capacity based on average**, i.e., without looking at the random nature, will result in high "loss" (patients not finding a recovery bed).

2. **Diminishing returns to increase in capacity**. The results are non-linear or not proportional. For instance, the number of additional beds needed increases much more when one needs a lower probability of "loss" (rejection), i.e., not finding an empty bed after procedure. This can be seen by moving up the table for the same column.

3. **Economies of scale**. On the other hand, as the demand gets higher, e.g., having more sedation cases, the relative increase in demand for recovery bed is lower. For instance, 10 beds have a 4.3% 'loss' with a demand of 6, while doubling both the capacity (to 20 beds) and demand (to 12) have a much lower 'loss' at 1.0%. This in a way, reflects economy of scale in a random environment.

Conclusion

The queueing model addresses the inherent randomness in patient arrival and procedure duration, or else the true demand would be underestimated and patient safety may be compromised. Furthermore, the study also revealed some insights into the relationship between demand, capacity and queues.

This work is done together with Sister Chia Yeow Peng and Sister Christina Tan Hwei Hian of TTSH Endoscopy Centre. We also like to thank them for kindly allowing the use of this case.

Upcoming 8th Operations Research Appreciation Course

Look out for our next run of ORAC on 28-29 Jan 10 where we will cover topics on queueing analysis, simulation, systems thinking, optimization and more. Please contact *palvannan_kannapiran@nhg.com.sg* for more information.