OPTIMIZATION

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22 February 2019

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The evidence behind your decisions

Outline

- Introduction
- Optimization Model
- An Illustrative Example using Optimization Model
- Two Studies in Health Service Research using Optimization

Introduction -- OR and Public Health

American Journal of Public Health
42 (10): 1306-1307 1952

OPERATIONS RESEARCH AND PUBLIC HEALTH

THE problem of translating theory into practice is usually beset with difficulty, even in the more exact natural sciences. In the past, this problem has suffered from neglect, perhaps because it was assumed that practical men could apply in practice any clearly stated theory, and needed no special agency to facilitate and accelerate this process.

Following the termination of hostilities, it was quickly realized that operations research methods and technics could have wide application to conditions and needs of peace in government, industry, and in the community in general.[4] Since the war

Clearly, here are exciting possibilities of interweaving theoretical insight with practical experience which no health worker can afford to overlook. While the

How valuable operations research will eventually be to public health remains to be seen. The surface of the problems presented by such interaction of research and policy decisions has barely been touched. All that can be done here is to draw attention to this important branch of scientific activity and to indicate its possible potential for public health.

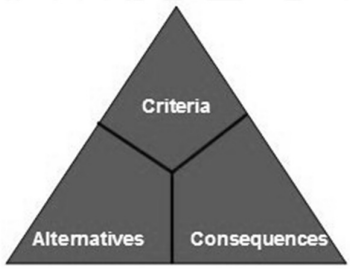
Introduction -- optimization applications in healthcare

- **Healthcare planning:** location selection (healthcare centres, emergency vehicles), capacity planning
- **Healthcare management and logistics:** patient scheduling, resource scheduling (nurse staffing, nurse scheduling, operating room and physician scheduling), hospital logistics
- **Healthcare practice:** disease diagnosis, treatment planning
- **Specialized and preventive healthcare:** organ donation and transplant, prevention of diseases

Introduction -- what is optimization?

Optimization (mathematical programming/optimization) is the *selection* of a *best* element with regard to some *criteria* from some set of available *alternatives*.

Optimization = search for the best solution



What is an Optimisation Problem?

1. You have some value(s) that you want to maximize / minimize [Objective(s)]
   - E.g. profit, loss, time, output etc

2. To do that, you need to make some decisions out of many possibilities [Decision Variables]
   - E.g. what stock to buy

3. And you typically has certain restrictions [Constraints]
   - E.g. budget, timeline, people

An optimization example

- Suppose we have a choice to treat different number of private and subsidised patients
- Revenue from a private patient / sub patient is $63 and $45 respectively
- Doctor consultation time for a private / sub patient is 35 min / 20 min respectively
- A private /sub patient takes 8 /10 min of nurses time
- The clinic has 7900 min of doctors' consultation time and 3000 min of nurses time
- From population profile analysis, we know that the number of sub patients will be between 180 and 250

Problem modelling using math notations

Decision:

numbers of private patients & subsidised patients, *x* , *y*.

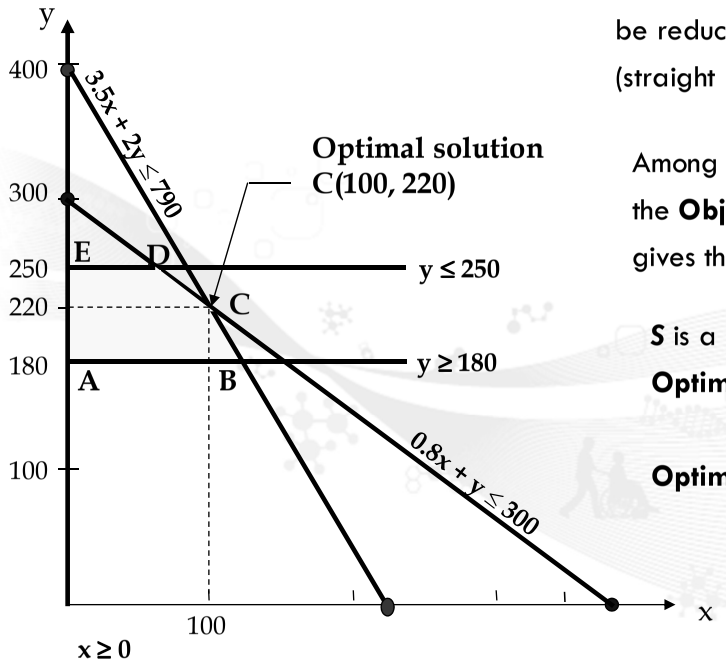
Objective:

maximise the total revenue, *maximise* $63x + 45y$

Constraints:

1. $8x + 10y \leq 3000$; (nurses time constraint)
2. $35x + 20y \leq 7900$; (consultation time constraint)
3. $y \leq 250$; (upper bound of number of subsidised patients)
4. $y \geq 180$; (lower bound of number of subsidised patients)
5. $x \geq 0$. (nonnegative number of private patients)

Solve the problem: graphical illustration



The x-y space marks the **possible choices,** to be reduced by the sets of **constraints** (straight lines)

Among the **feasible region S,** the point where the **Objective** function reaches the maximum gives the **optimal** point

S is a polyhedral set, i.e., ABCDE

Optimal solution:

x=100, y=220

Optimal value:

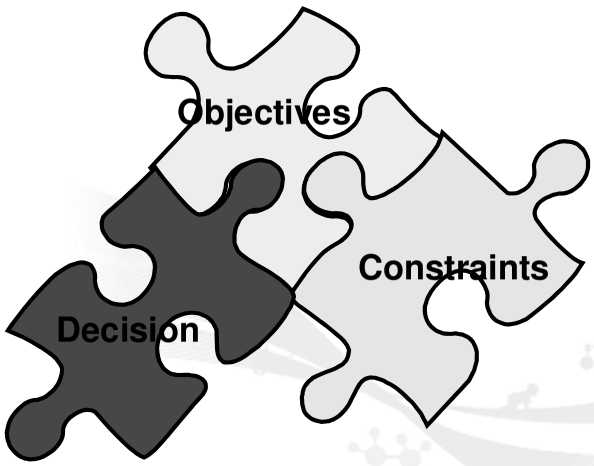
63x100 + 45x220 = $16,200

Insights

- Constraints, decision variables may be multiple
- The optimal solution can be found at the intersection of some constraints (called corner points).

  + Hence at optimality, some constraints are binding and some might not.
- The optimal solution can be unique, finite/infinite, or even does not exist.
- Adding constraints to a problem shall not improve the objective value.

Optimization Framework: Decision, Objectives, Constraints



I. Decision (Do what)
   + Basic option to be chosen (who, what, when, where)

II. Objectives (Optimise what)
   + Criteria to maximize or minimize (utilization or cost, etc)

III. Constraints (Feasibility)
   + Policy (subsidy, private)
   + Legacy (culture now = culture before)
   + Physical limits (usable space ≤ max space)
   + Demand (meet current operating load)
   + Conservation (inflow = outflow)

Optimization Model

Minimize (or maximize) an *objective function*
with respect to some *decision variables*
subject to some *constraints*

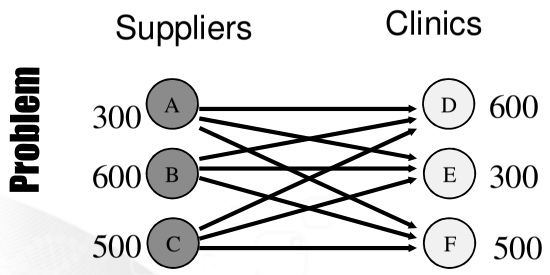
Example

$$\begin{aligned} \min \quad & 2x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 - 10x_2 + 7 \leq 0, \\ & x_1 + 4x_2 - 5 = 0. \end{aligned}$$

Most common optimization models

Optimization models	Linear Programming (LP)	Integer Programming (IP)	Mixed Integer Programming (MIP)
Objective	Linear function	Linear function	Linear function
Constraints	Linear functions	Linear functions	Linear functions
Decision variables	Nonnegative	Nonnegative; All variables are integers	Nonnegative; Some variables are integers

Medical items delivery plan with the least total shipping cost



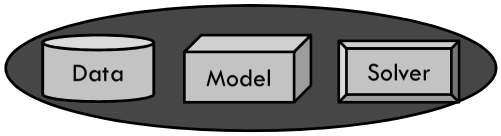
	D	E	F
A	$0	$4	$1
B	$1	$6	$3
C	$3	$7	$6

Delivery cost matrix (per unit)

Declarative

- State the data
- State decision variables, objective, constraints
- Formulate the model
- Solve the model

Structure Problem



Data: number of suppliers; number of clinics; delivery cost per unit;

capacity of each supplier; demand from each clinic

Decision variables:

Number of items shipped from suppliers to clinics

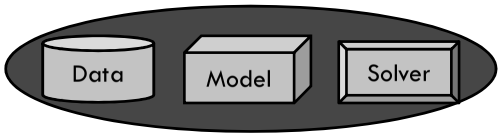
Objective:

minimise the total delivery cost

Constraints:

1. For each supplier, quantity of items **>=** quantity shipped;
2. For each clinic, total quantity of items received **=** its demand;
3. Quantities of items in delivery **>=0**.

Solve optimization models

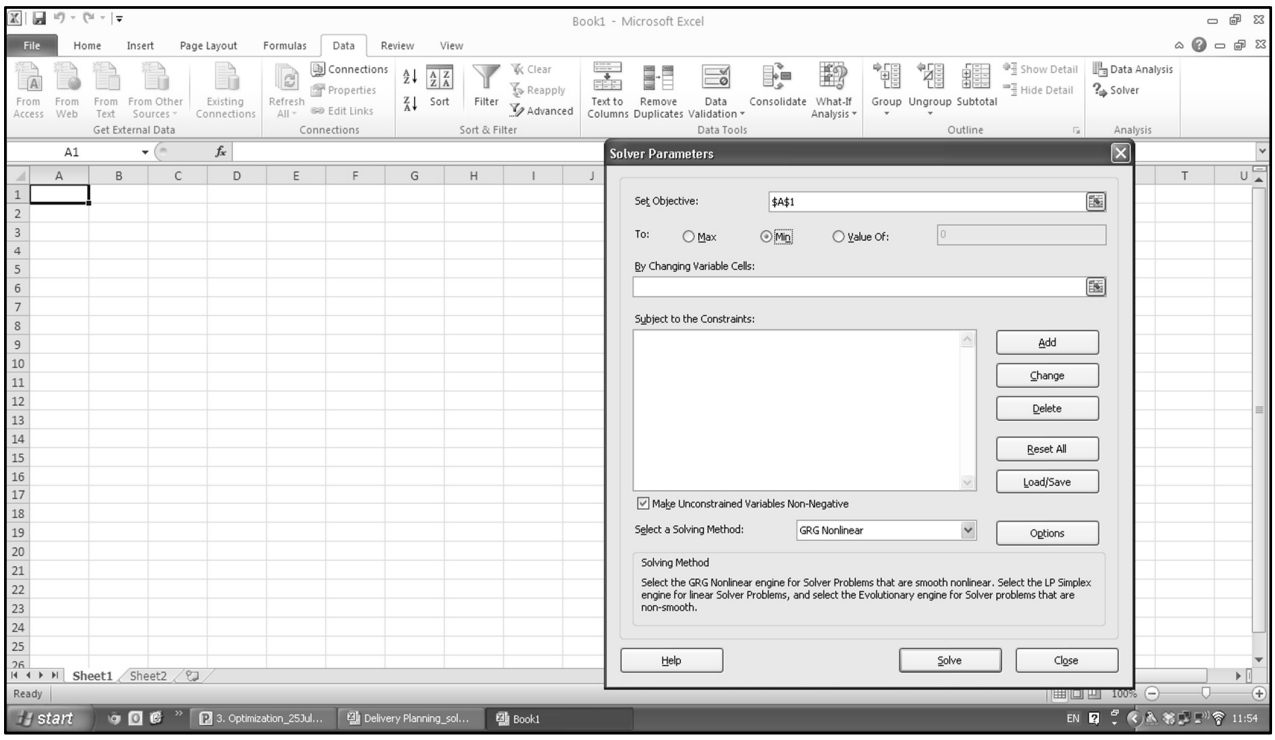


Solve optimization models:

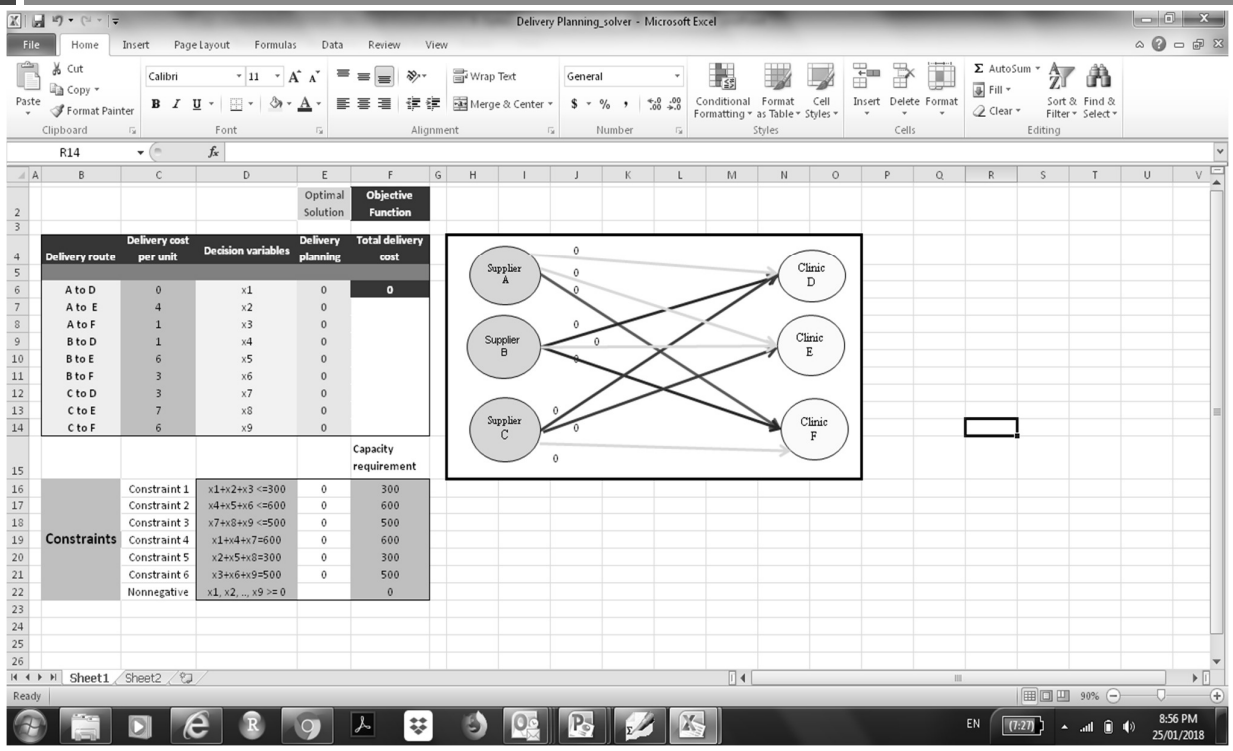
1. MS Excel optimization solver (applicable to small scale problems)
2. Other commercial software:

   AMPL, CPLEX, Gurobi, LINDO API, LINGO, Maple, Mathematica, MATLAB, OPL Studio

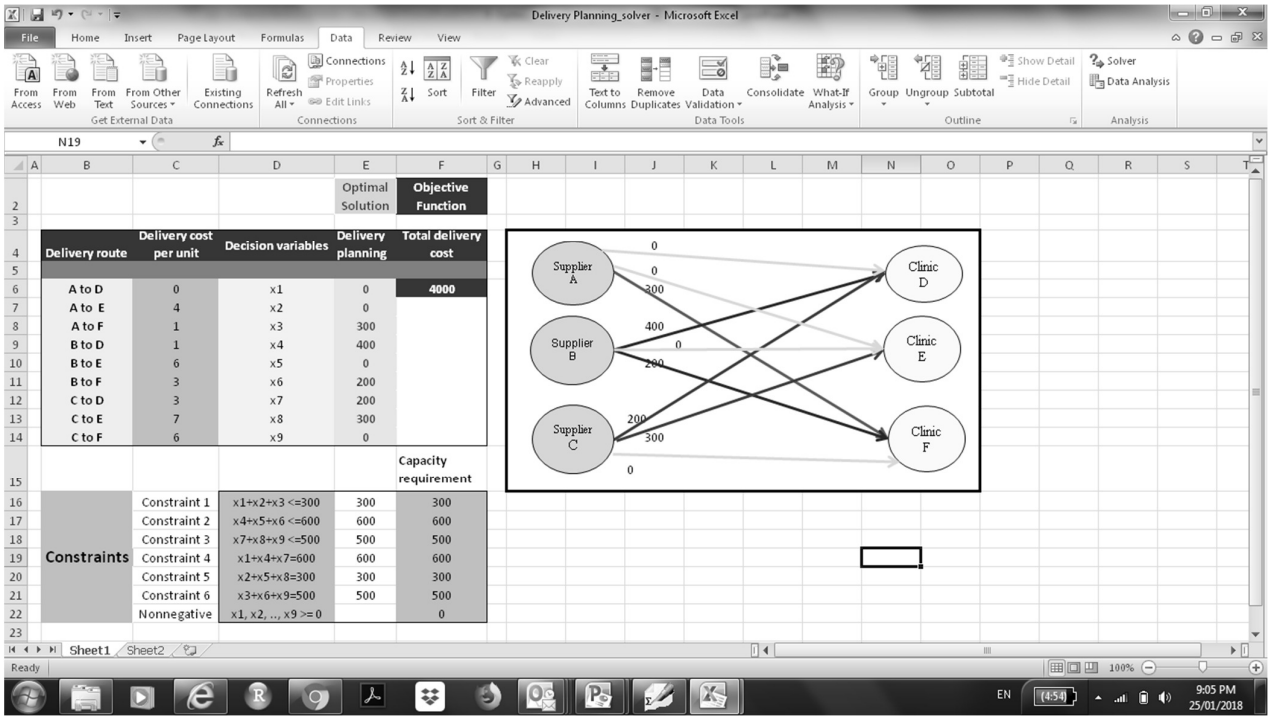
Excel Optimization Solver: least cost transport plan



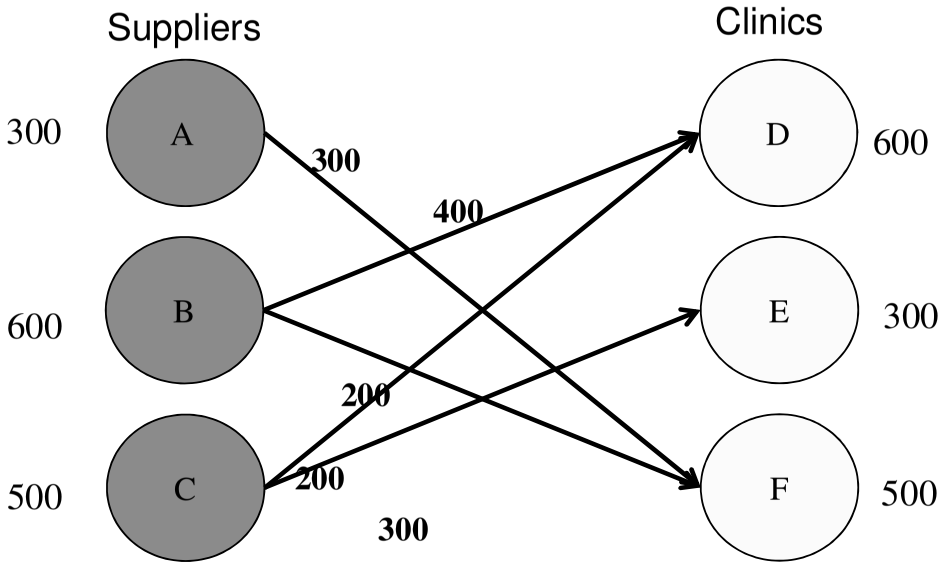
Excel Optimization Solver



Excel Optimization Solver



Optimal delivery plan (the minimum delivery cost: **$4,000**)



Study 1: shift capacity planning for nurse staffing at ED

Background:

- Workload in ED changes by day-of-week and time-of-day. Patients flow into different areas of ED depending on their needs.
- The ED nurses are one critical component in the management of the patients. The nurses have different skill sets and are deployed at different functional areas of ED, and work in shifts.
- Matching the workload with the staffing can thus be a complex issue.

Study Objective:

- Develop mathematical model for nurse capacity planning

Shift capacity planning for nurse staffing at ED

Areas at ED:

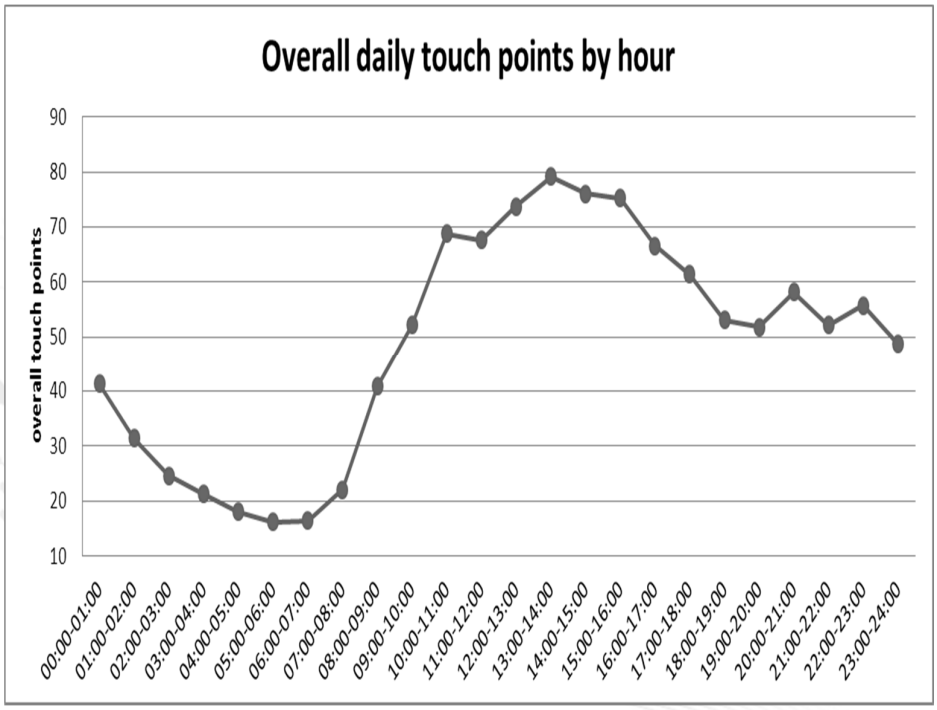
- Triage; Consult; AUC; Resus; Decon; EDC Obs; EDX consult, EDX Obs & EDX Iso; EDTC.

Shifts at ED:

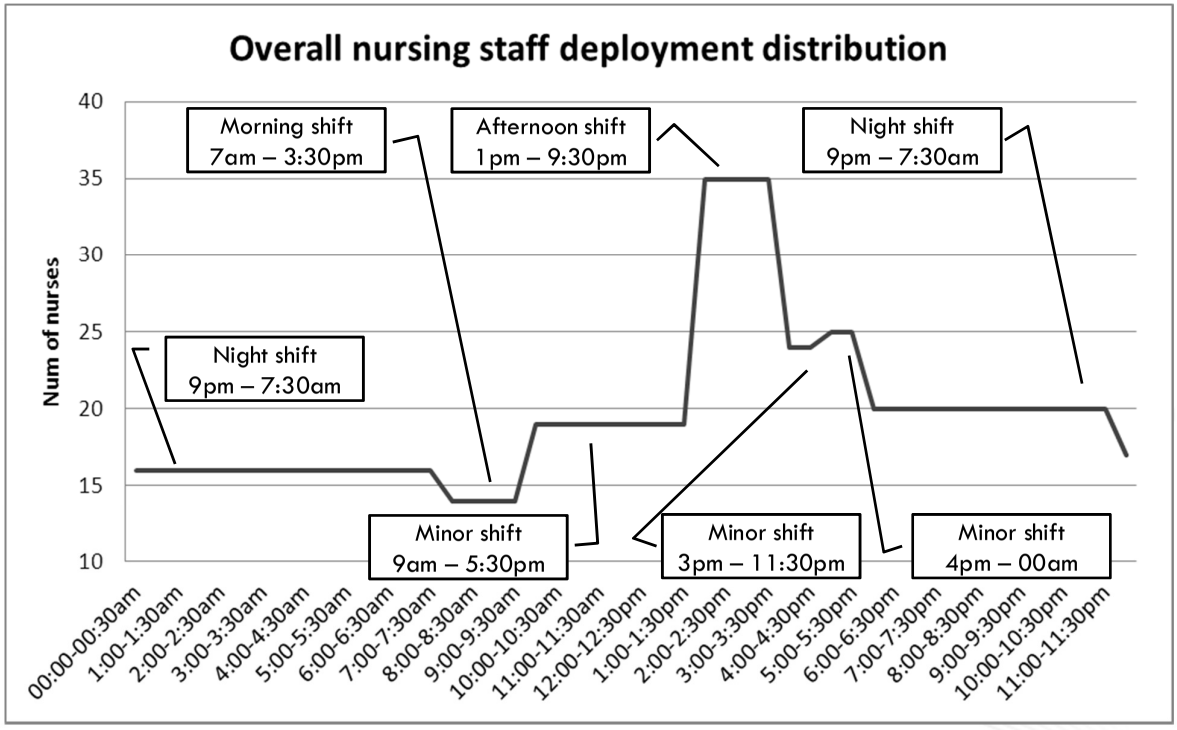
- *3 major shifts*:
  - ❑ morning shift (7am-3:30pm)
  - ❑ afternoon shift (1pm-9:30pm)
  - ❑ night shift (9pm-7:30am)
- *3 minor shifts:*
  - ❑ minor shift 1 (9am-5:30pm)
  - ❑ minor shift 2 (3pm-11:30pm)
  - ❑ minor shift 3 (4pm-00am)

Daily demand/touch points

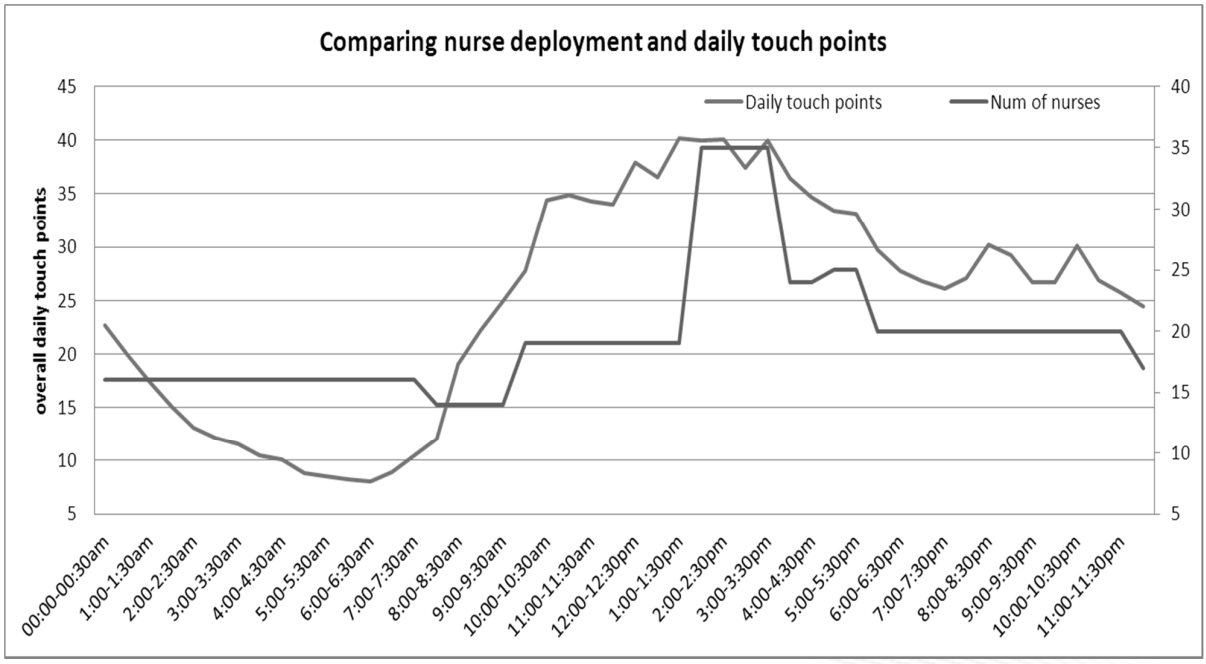
Area	Daily touch Points
Triage	449
Consult	205
AUC	192
EDC Obs	169
EDX Consult	76
EDX Obs	41
Resus	26
Decon	12
EDX Iso	3



Overall nursing staff deployment



Comparison of touch points and nursing staff deployment



Optimization model for shift capacity planning

Assumption:

- Current total manpower supply is unchanged
- Current shift times are unchanged

Objective:

- To balance the overall workload during a day, i.e., to minimize the mean deviation of workload during the 48 half an hour intervals of a day

Decision variables:

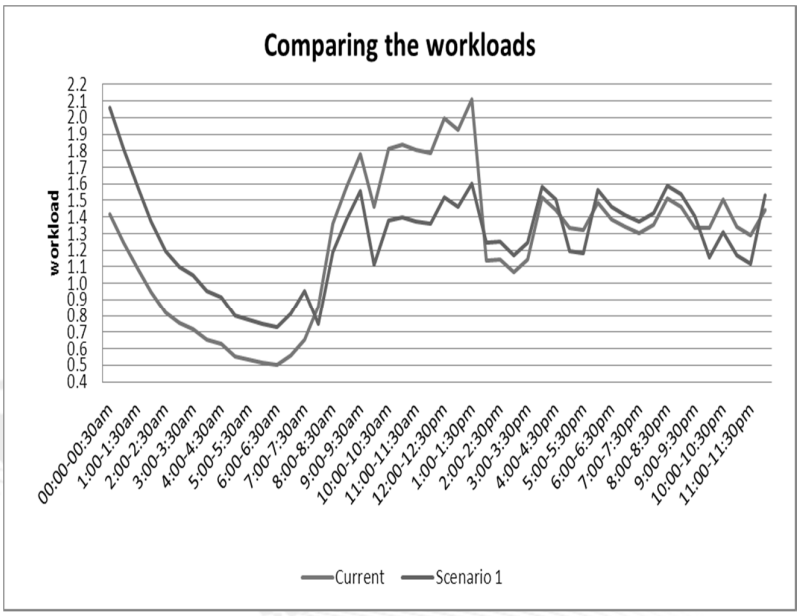
- Numbers of nursing staff of 6 shifts

Scenarios for shift capacity planning

- **Scenario 1:**
  - no constraint on the number of nurses for each shift;
  - the maximum workload <= current maximum workload.
- **Scenario 2:**
  - current nurse capacity of major shifts may decrease/increase *x* nurses; where x= 1, 2, 3, 4, 5, 6.
- **Scenario 3:**
  - number of nurses of night shift is unchanged.

Results of scenario 1

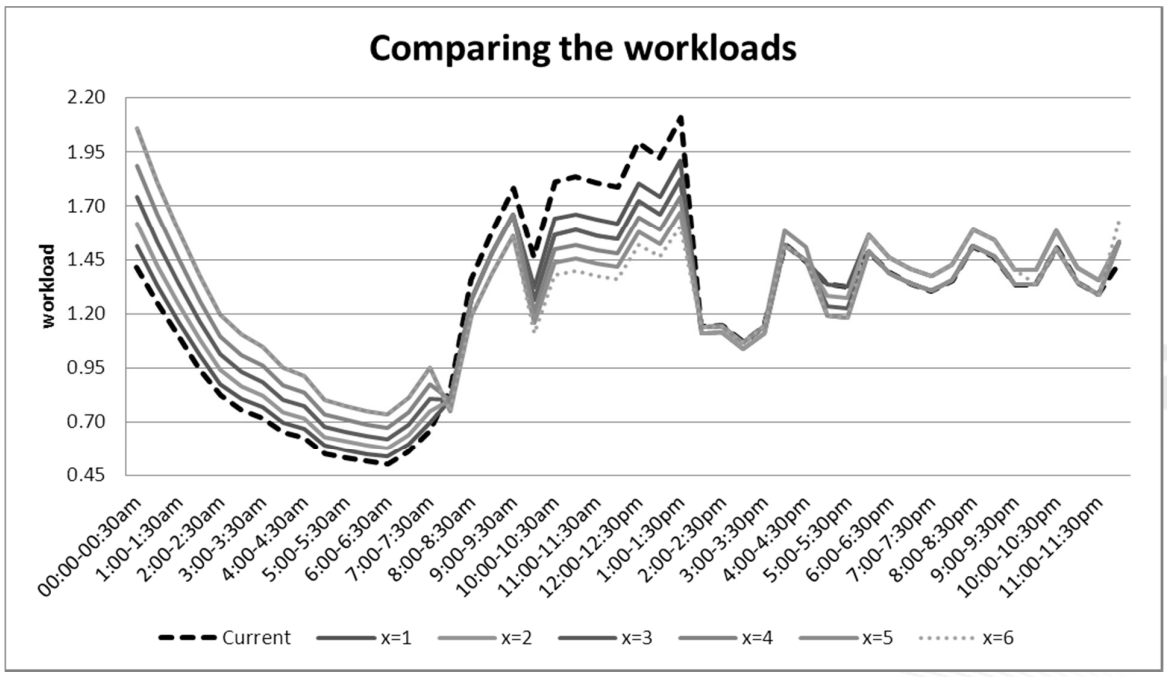
	Current	Scenario 1
Morning shift	14	**16**
Afternoon shift	16	**7**
Night shift	16	**11**
Minor shit 1	5	**9**
Minor shift 2	3	**7**
Minor shift 3	1	**5**
Total capacity	55	55
Mean of workload	1.25	1.28
Mean deviation	0.34	0.23
Mean deviation reduction		**32.4%**
max WL - min WL	1.61	1.33



Results of scenario 2

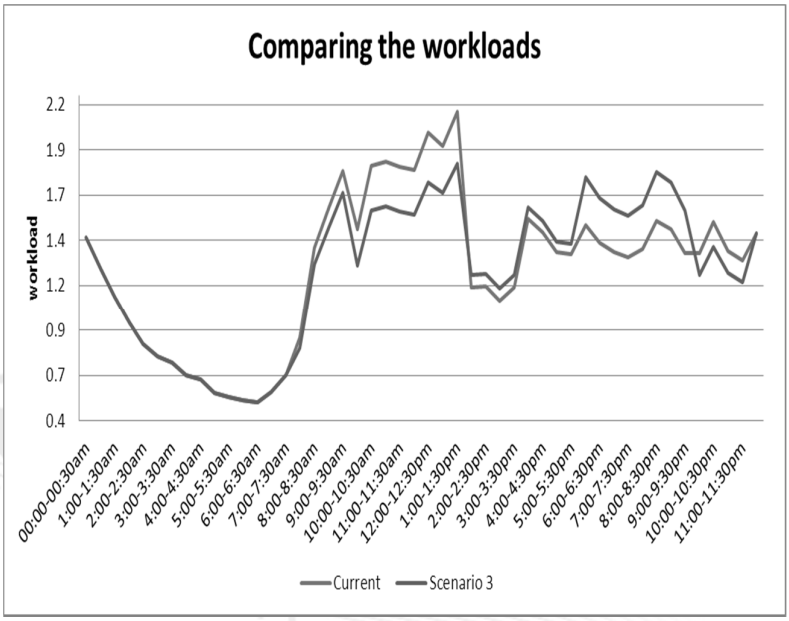
	Current	x=1	x=2	x=3	x=4	x=5	x=6
Morning shift	14	15	15	15	15	16	16
Afternoon shift	16	15	14	13	12	11	10
Night shift	16	15	14	13	12	11	11
Minor shit 1	5	6	7	7	8	8	9
Minor shift 2	3	3	3	4	4	4	5
Minor shift 3	1	1	2	3	4	5	4
Total capacity	55	55	55	55	55	55	55
Mean workload	1.25	1.25	1.25	1.26	1.27	1.28	1.28
Mean deviation	0.34	0.32	0.30	0.27	0.25	0.24	0.24
Mean deviation reduction		5.8%	13.0%	21.4%	26.1%	30.1%	28.9%
max WL - min WL	1.61	1.37	1.25	1.20	1.22	1.33	1.33

Results of scenario 2



Results of scenario 3

	Current	Scenario 3
Morning shift	14	15
Afternoon shift	16	11
Night shift	16	16
Minor shit 1	5	7
Minor shift 2	3	5
Minor shift 3	1	1
Total capacity	55	55
mean of workload (7:30am-00am)	1.47	1.46
mean deviation (7:30am-00am)	0.21	0.19
mean deviation reduction (7:30am-00am)		11.5%
max WL - min WL (7:30am-00am)	1.25	1.02



Study 2: PKPD (pharmacokinetic pharmacodynamic) modeling -- optimal drug dosing for patients with Graves' disease

Background:

- Graves' disease is an autoimmune disorder, the most common cause of hyperthyroidism
- Each year, 0.02% - 0.05% of the population will be newly diagnosed with Graves' disease[1]
- Three treatments for Graves' disease[2]
  - **Antithyroid drugs** (ATD): to block the production of thyroid hormones
  - **Radioactive iodine** (RAI): to destroy the thyroid gland
  - **Thyroidectomy**: to remove the thyroid gland surgically
- Multiple factors, e.g., patient preference & risk of relapse, are involved in determining the treatment modality

Reference:
1. T. Smith, L. Hegedus. Graves' disease. ***The New England Journal of Medicine***. 2016, 375:1552-1565.
2. R. S. Bahn. Graves' disease: a comprehensive guide for clinicians. Springer New York, 2015.

Background

- ATD treatment allows patient to avoid lifelong thyroxine replacement, if successful in bringing about remission[3]
- Two regimens to adopt ATD treatment
  - **Titration regimen**: adjust ATD dosage based on thyroid tests such that the patient achieves euthyroidism at the lowest dosage of ATD
  - **Block-and-replace regimen**: give patients a high dosage of ATD to suppress production of thyroid hormones
  - Both have comparable effectiveness. Titration regimen has less adverse effects to patients but with more frequent clinician visits[4]

Reference:
3. P. Laurgerg, A. Krejbjerg, S. Andersen. Relapse following antithyroid drug therapy for Graves' hyperthyroidism. **Current Opinion in Endocrinology Diabetes and Obesity**. 2014, 21:415-421.
4. B. Vaidya, et al. Block and replace regime versus titration regime of antithyroid drugs for the treatment of Graves' disease: a retrospective observational study. ***Clinical Endocrinology***. 2014, 81: 610-613.

Objective of the study

Current practice of titration of ATD treatment:

- based on thyroid function trend
- arbitrary and time-consuming, requiring multiple clinic monitoring visits before an optimal dose is found

Objective:

- To develop a personalized medicine model that facilitates optimal drug dosing via the titration regimen

Research Design and Method

- Analyse patients' thyroid function tests (TFT) data, consisting of drug dosage, time period and serum free thyroxine (FT4)
- Use ordinary differential equation (ODE) modeling to describe the dynamic behavior of FT4 concentration
- Develop an optimization model to determine parameters of synthesis rate, decay rate and IC_{50} in FT4 equation, with each patient's data
- Establish the closed-form time- and dose-dependent solution, which allows explicit estimates of personalized predicted FT4

Descriptive statistics of TFT data, *N=49*

Item	Review visit (time)	FT4 value (pmol/L)	Review interval (day)
Mean	6.9	20.7	73.5
Standard Deviation	4.7	16.0	33.9
Median	5	15	70
Minimum	2	1	5
Maximum	24	91	210

- Data sets: TFT data with first x-visit; with at most x-visit, x=3, 4, 5
- Parameters: synthesis rate; decay rate; IC_{50}
- Estimated parameters are acceptable if the predicted FT4 value is within a tolerance of 4.5 pmol/L

Estimation accuracy rates with different data sets, *N*=49

Data set	Number of patients	Number of patients meeting the tolerance	Estimation accuracy rate
First 3-visit	48	37	77.1%
At most first 3-visit	49	38	77.6%
First 4-visit	36	27	75.0%
At most first 4-visit	49	36	73.5%
First 5-visit	31	26	83.9%
At most first 5-visit	49	35	71.4%

FT4 estimates for given dosages and review periods

Dose (mg)	Time period (day)	Predicted FT4 (pmol/L)
3	35	19.4
	56	17.4
	63	17.1
8	35	18.1
	56	16.2
	63	15.9
13	35	16.9
	56	15.2
	63	14.9

Optimal dose estimates for targeted FT4 values & review time periods

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Target FT4 value (pmol/L)	Time period (day)	Dose (mg)
12	35	44.1
	49	36.6
	63	32.1
15	35	22.3
	49	15.4
	63	12.0
17	35	12.7
	49	6.3
	63	3.5

Predicted FT4 curves for different drug dosages (initial FT4 value = 75 pmol/L)

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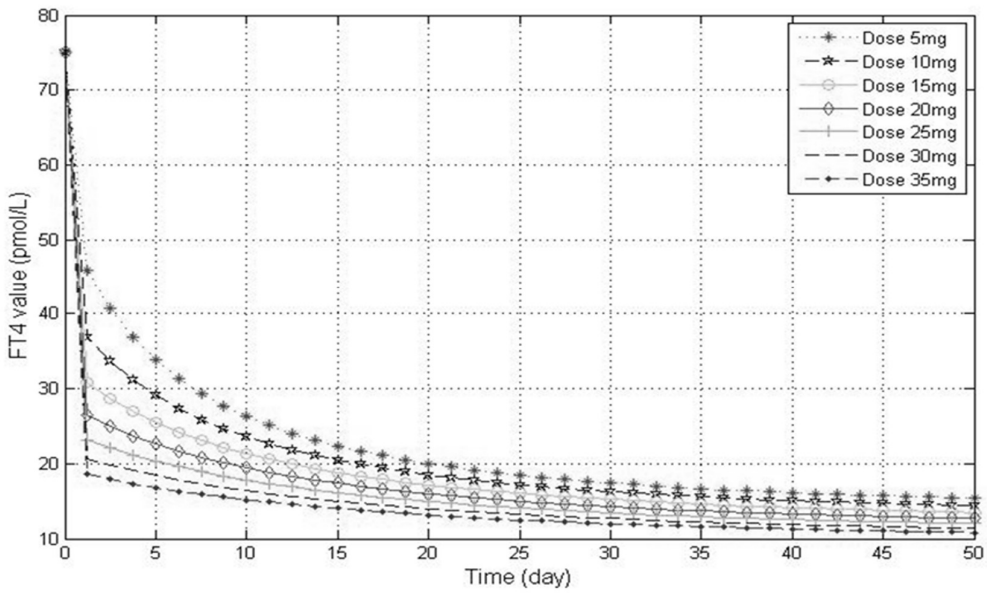
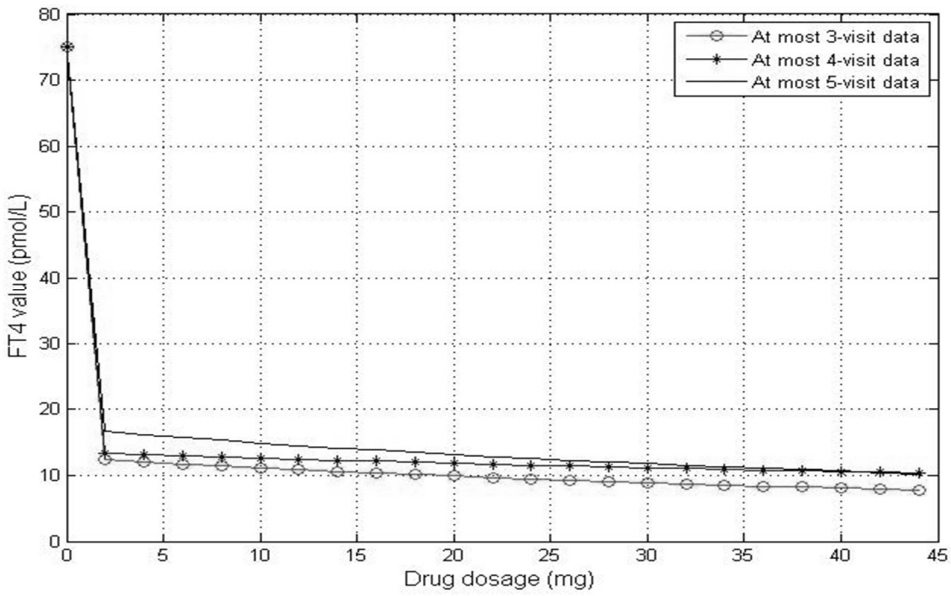


- The higher the dose, the steeper the FT4 curve in the decline phase
- Patients with the lower drug dose would take a longer time to achieve the same FT4 target

Predicted FT4 curves with various doses in 42 days based on different data sets (initial FT4 value = 75 pmol/L)



- Patients may take lower drug doses to achieve the same desired FT4 target when the FT4 approximation was based on first fewer visit data

Summary

1. We introduced ODE to describe the dynamic behavior of FT4 concentration
2. We developed an optimization model to determine parameters such as synthesis rate, decay rate, IC_{50} with the TFT data of each patient
3. We derived the closed-form solution of the ODE and explicit personalized estimation of FT4
4. We derived favorable accuracy rate (**77.1%, 75.0%, and 89.9%**) of predicted FT4 using data from the first 3, 4 and 5 visits respectively, compared against actual FT4 data within a tolerance of 4.5 pmol/L.

Other examples using optimization

1. Facility location:
   + Where should we place the service points, e.g. hospitals, Polyclinics, to provide sufficient service and minimise distance travelled?
2. Roster:
   + Staff roster for a team, e.g. doctor, nurse, etc
3. Bed allocation:
   + Given current inpatient beds
   + How many beds should be given to each department?
   + Where should the beds be?
4. Inventory planning:
   + Inventory (e.g. gloves, drugs) that needs to be stored, used, purchased
   + How many to buy, when to buy?

Applications in healthcare have extended to, for instance, using

- stochastic optimisation modelling for ambulance allocation problems
- hybrid models incorporating queueing & robust optimisation for bed assignment & patient scheduling
- Markov models for managing patient admission process & patient flow
- Markov decision process based on models & machine learning for health screening, disease classification & prediction, personalized medicine
- simulation models such as discrete event simulation & microsimulation for predicting patient disease progression, the trajectory of population health conditions, the trend of system costs of a population

Some references in healthcare

1. Recent modeling and analytical advances in hospital inpatient flow management. Cornel University, November 2018 (Jim Dai & Pengyi Shi)
2. Applying gravity model to predict demand of public hospital beds. *Operations Research for Health Care*. https://doi.org/10.1016/j.orhc.2017.09.006 2017 (Teow KL et al)
3. A robust optimization model for managing elective admission in a public hospital. *Operations Research* **63**, 1452-1467, 2015 (Meng FW et al.
4. Intelligent analysis of acute bed overflow in a tertiary hospital in Singapore. *Journal of Medical Systems* **36**, 1873-1882, 2012 (Teow KL et al)
5. Polyclinic sites selection. *HSOR project*, 2011 (Teow KL et al)
6. Optimising package size in the automated dispensing machine of outpatient pharmacy. *HSOR Annual Report 2012* (Teow KL et al)
7. Impact of workload on service time and patient safety: an economic analysis of hospital operations. *Management Science* **55**, 1486-1496, 2009 (Diwas S et al)
8. Mixed integer programming approaches to treatment planning for Brachytherapy – application to permanent prostate implants. *Annals of Operations Research* **119**, 147-163, 2003 (Lee EK et al)
9. Optimization of operating room allocation using linear programming techniques. *Journal of American College of Surgeons* **197**, 889-895, 2003 (Kuo PC et al)

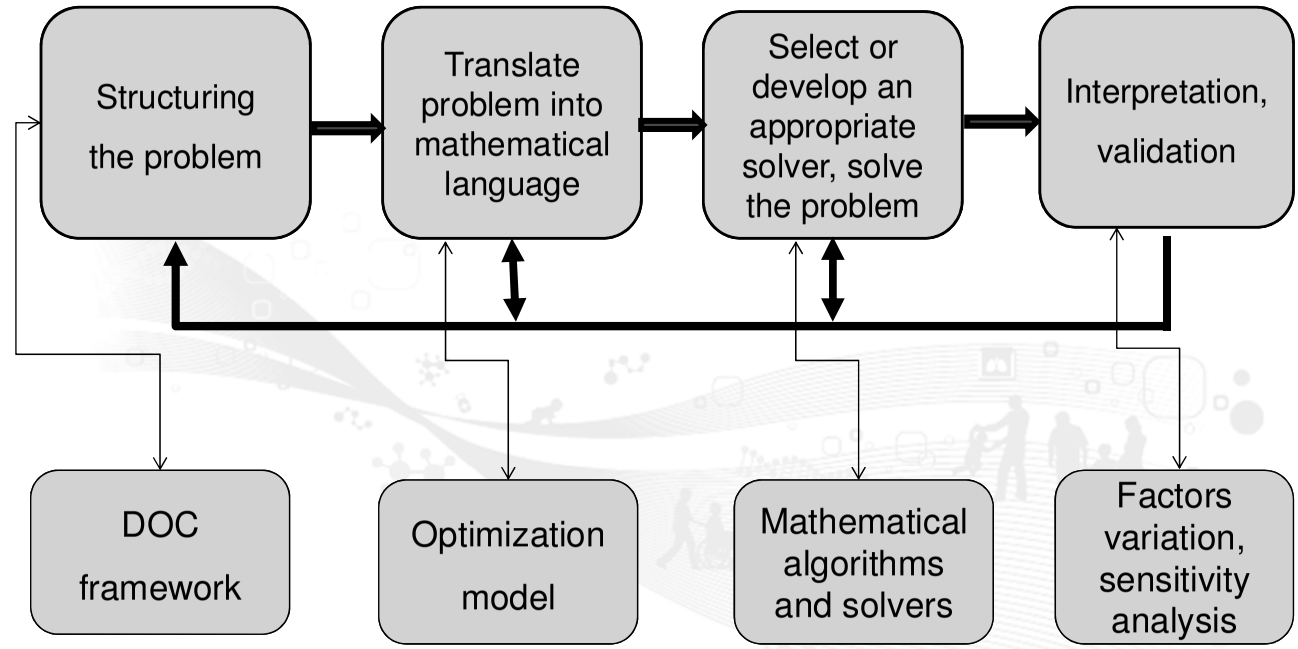
Summary -- key points in optimization

- Choice of best strategy
- Choice of best estimate
- Choice of best design parameters
- Choice of best combinations of scenarios

Criteria for Optimization:

- Performance index
- Value function, e.g., time, distance
- Cost function
- Fitness function

Summary -- optimization modeling system



Appendix

Optimization model for shift capacity planning

Parameters

- C: total capacity of nursing staff
- T_j: j-th time interval of a day, $j = 1, \ldots, J$, where the first time interval starts at 00:00 and the last one ends at 24:00, denoted by an index set $\mathcal{T} := \{\mathrm{T}_1, \ldots, \mathrm{T}_J\}$
- C_j: minimum number of nursing staff during time interval T_j, $j = 1, \ldots, J$
- d_j: adjusted demand during time interval T_j, $j = 1, \ldots, J$
- $\bar{x}_j$: current number of nursing staff on duty during time interval T_j, $j = 1, \ldots, J$
- ρ: minimum ratio of the number of nursing staff to the demand amongst J time intervals, i.e., $\rho := \min\{\frac{\bar{x}_j}{d_j} : j = 1, \ldots, J\}$
- $\mathcal{K}_1$: index set of major shifts, denoted by $\mathcal{K}_1 :=$ {morning, afternoon, night} $= \{\mathrm{M}_1, \mathrm{M}_2, \mathrm{M}_3\}$
- S: maximum number of possible minor shifts of a day, denoted by $\mathcal{K}_2 := \{\mathrm{M}_4, \ldots, \mathrm{M}_{S+3}\}$
- $\mathcal{K}$: index set of all possible shifts (including both major and minor shifts) of a day, that is, $\mathcal{K} := \mathcal{K}_1 \cup \mathcal{K}_2 = \{\mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_{S+3}\}$
- γ: maximum number of minor shifts to be operated in ED
- l_i: lower bound on number of nursing staff in shift $\mathrm{M}_i \in \mathcal{K}$, $i = 1, \ldots, S+3$
- u_i: upper bound on number of nursing staff in shift $\mathrm{M}_i \in \mathcal{K}$, $i = 1, \ldots, S+3$

Optimization model for shift capacity planning

Decision variables

- x_i: number of nursing staff in shift M_i, $i = 1, \ldots, S+3$
- y_j: number of nursing staff in time interval T_j, $j = 1, \ldots, J$
- z_i: 0-1 binary variable, $i = 4, \ldots, S+3$. For each $\mathrm{M}_i \in \mathcal{K}_2$,

$$z_i := \begin{cases} 1, & \text{if minor shift } \mathrm{M}_i \text{ is selected for capacity planning,} \\ 0, & \text{otherwise.} \end{cases}$$

Optimization model for shift capacity planning

$$
\begin{aligned}
\min \quad & \frac{1}{J}\sum_{j=1}^{J}\left|\frac{y_j}{d_j} - \frac{1}{J}\sum_{k\in\mathcal{T}}\frac{y_k}{d_k}\right| \\
\text{s. t.} \quad & \mathbf{1}'x \le C, \\
& l \le x \le u, \\
& y = A'x, \\
& y_j \ge \rho d_j, \quad j = 1,\ldots,J, \\
& y_j \ge C_j, \quad j = 1,\ldots,J, \\
& z_1 = z_2 = z_3 = 1, \\
& \sum_{i=4}^{S+3} z_i \le \gamma, \\
& x_i \le M_0 z_i, \quad i = 4,\ldots,S+3, \\
& z_i \le x_i, \quad i = 4,\ldots,S+3, \\
& x_i \in Z^+,\ z_i \in \{0,1\},\ i = 1,\ldots,S+3,\ \ y_j \in Z^+,\ j = 1,\ldots,J.
\end{aligned}
$$